CORPORATE FINANCE FOR LONG-TERM VALUE

Chapter 19: Options

Chapter 19: Options

The BIG Picture

- Options are contracts that give the owner the right to buy or sell a security at a pre-specified price

Discussion

- Financial options can be used to deal with uncertainty (e.g. declining price)
- A real option is the opportunity to make a particular business decision, exemplifying the value of flexibility
- Real options on F can have E or S drivers: payoff in terms of F, but with E or S as the underlying values
- Companies have a lot of put options against society, but awareness of them is low: this calls for integrated value expressed in real options

Financial options

- Financial options are option contracts that give their owners the right to sell or buy a security from the writer of the contract at a *specified price* ← **exercise / strike price**
- The two parties to the contract have opposite positions:
  - The buyer (owner) is **long** the option and has a **right** (not obligation) to buy or sell
  - The seller (writer) is **short** the option and has the **obligation** to fulfil the contract
- The owner pays a price for the option to the seller, known as the **option premium**
- Two types of options:
  - **Call options**, which gives the owner the **right to buy** a security
  - **Put options**, which gives the owner the **right to sell** a security

Security can be:
company stock, exchange rate, interest rate, commodity, etc.

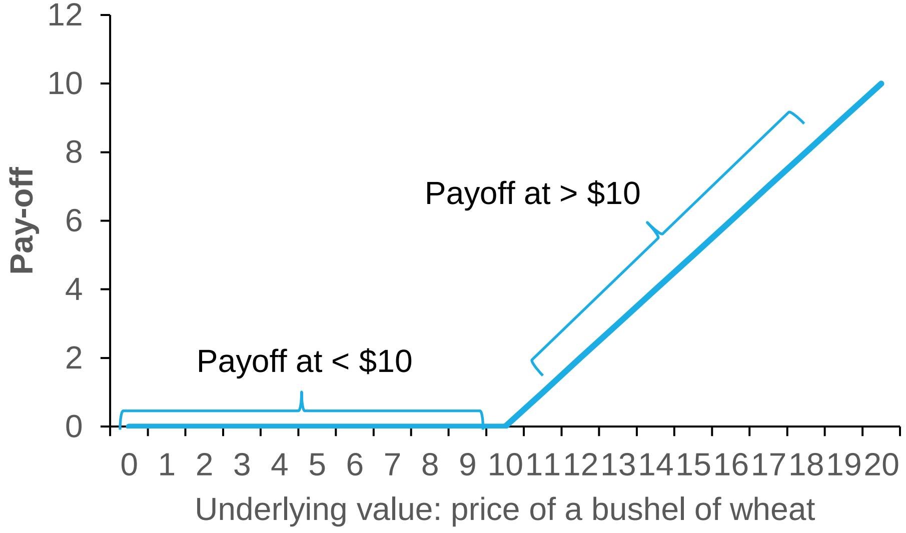
Call option – long

- Consider the example of a call option on a bushel of wheat, with an exercise price of $10
- The underlying value is the price of the bushel of wheat
  - The payoff is $0 for every price < $10
  - For every price > $10, the payoff is:

    price – exercise price
- The formula for a long position in a call is:

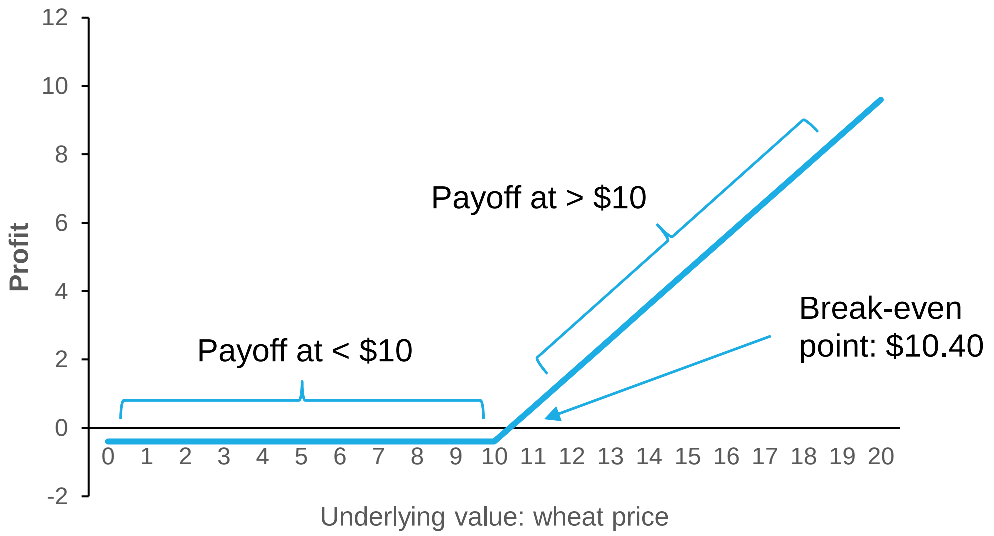
$$C = \max(S - K, 0)$$

Where S is the underlying value and K the exercise price



Call option – long (with premium)

- The longer the maturity of the call, the higher the probability that the underlying value will at some time exceed the strike price and the higher its value
- Since it has value, investors will be willing to pay a price for it, called a premium
- Suppose the premium for a bushel of wheat is $0.40, then the payoff structure becomes →
- Note that premiums are not fixed: they move with the price of the underlying security



- The buyer pays the premium to the seller at the time both parties enter into the contract, at which time the premium payment is fixed

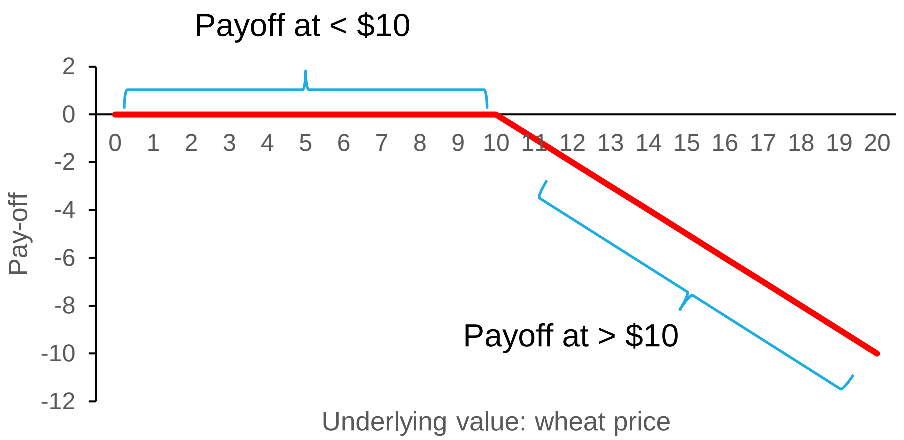
Call option – short

- The seller, who writes the call, has the exact opposite payoff profile to the buyer
  - The payoff is $0 for every price < $10
  - For every price > $10, the payoff is:

    - (price – exercise price)
  - The formula for a short position in a call is:

$$-C = -\max(S - K, 0)$$

Where S is the underlying value and K the exercise price



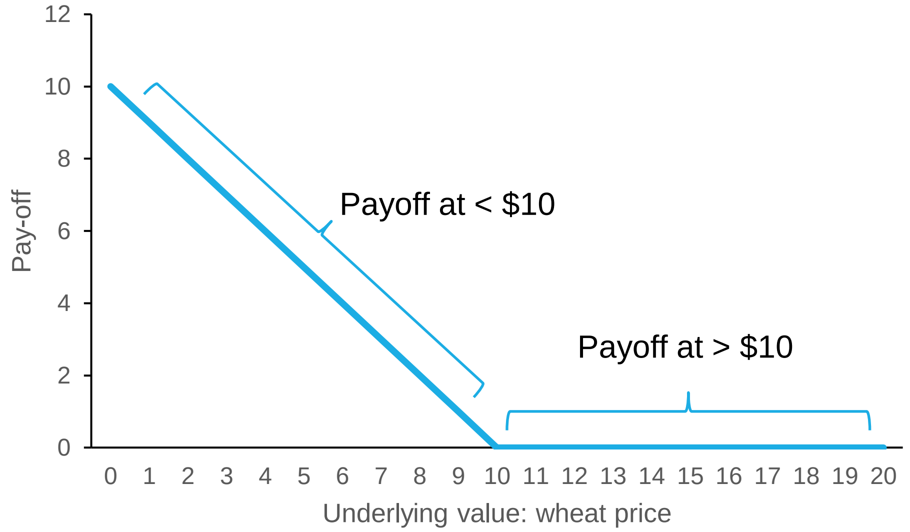
Put option – long

- A put option gives the owner the right to sell a security
  - For every price < $10, the payoff is:

    exercise price – price
  - The payoff is $0 for every price > $10
  - The formula for a short position in a call is:

$$P = \max(K - S, 0)$$

Where S is the underlying value and K the exercise price

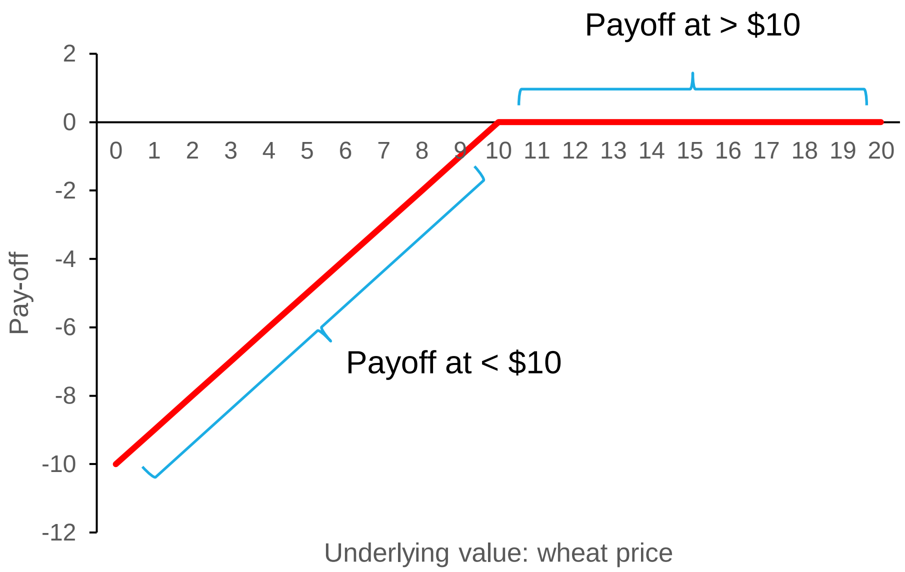


Put option – short

- The seller, who writes the put, has the exact opposite payoff profile versus that of the buyer
  - For every price < \$10, the payoff is:
    - (exercise price – price)
  - The payoff is \$0 for every price > \$10
  - The formula for a short position in a call is:

$$-P = \max(K - S, 0)$$

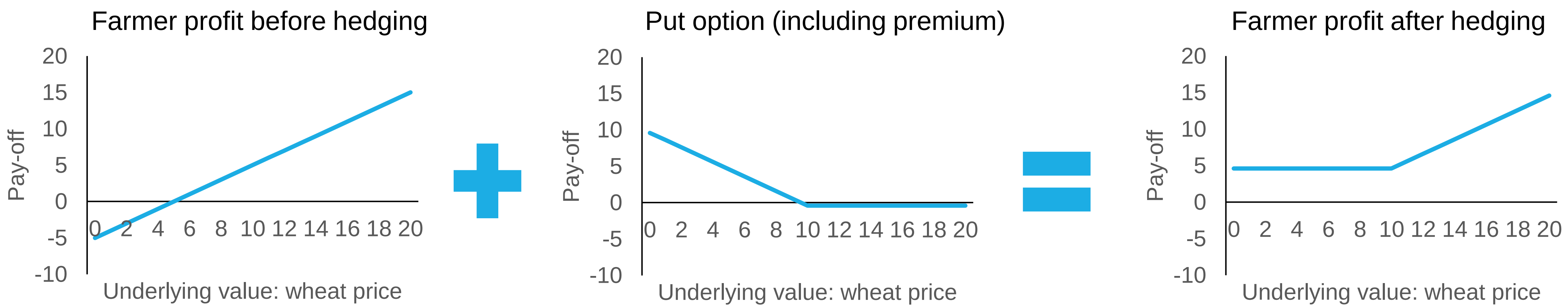
Where S is the underlying value and K the exercise price



Combinations of options & hedging

- In practice people can have composite exposures: they may use options to hedge their exposures
- Example: a farmer has a profit which depends on the wheat price
  - Costs are $5 per bushel, so profit is: wheat price - $5 (left graph)
  - By buying the put option with exercise price $10, the farmer gets protection against losses (middle graph)
  - The farmer's payoff: profit before hedging (S - $5) plus the put's payoff (max[K-S,0]) minus the put's premium:

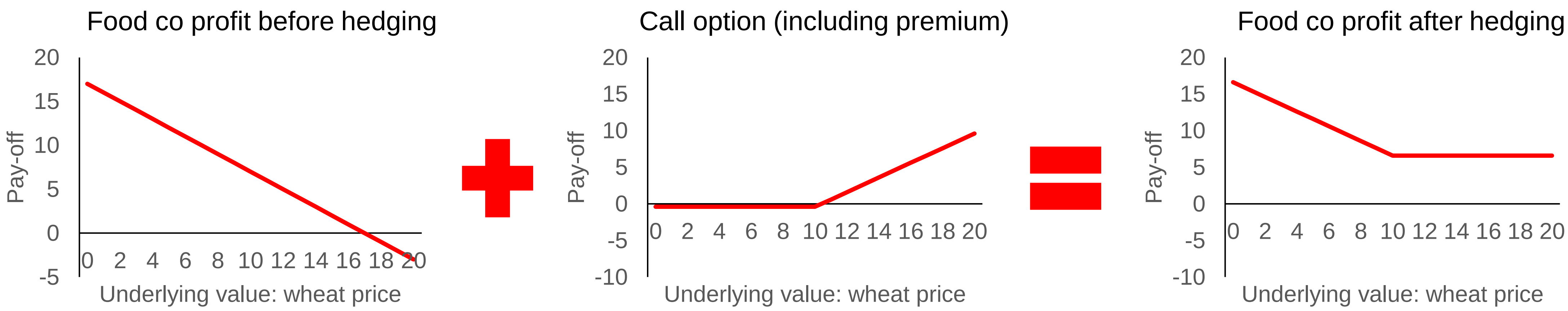
    (S - $5) + max(K-S,0) - $0.40 → at all prices below $10, profit is locked in at $4.60 ← **farmer buys protection at a cost of $0.40**



Combinations of options & hedging

- A producer of packaged food faces an (almost) opposite exposure
- The price per bushel hurts its profits since the food company needs to buy wheat for its production
  - Profit per bushel: $17 – price per bushel (left graph)
  - By buying a call option with exercise price $10, the food company gets protection against losses (middle graph)
  - The food company’s payoff: profit before hedging ($17 - S) plus the call’s payoff (max[S-10,0]) minus the put’s premium:

    ($17 - S) + max(S-10,0) - $0.40 → at all prices above $10, profit is locked in at $6.60 ← **company buys protection at a cost of $0.40**

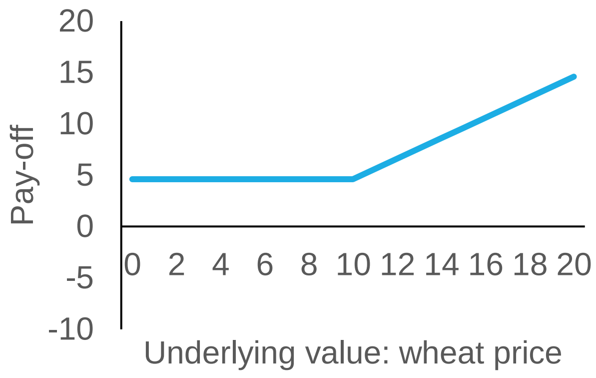


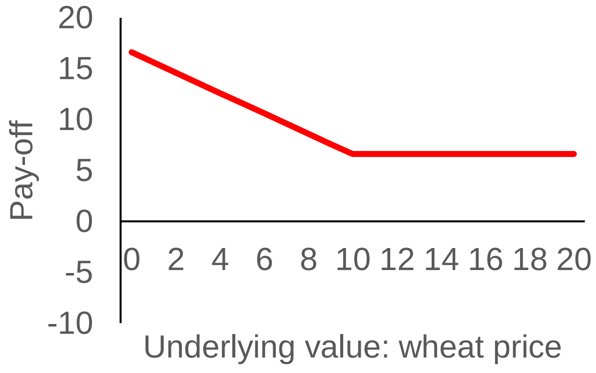
Put-call parity

- The exposure of the farmer looks like the call option of the producer, but at a higher level (including a bond)
- The exposure of the producer looks like the put option of the farmer, but at a higher level (including underlying)
- From this, the assumption follows:

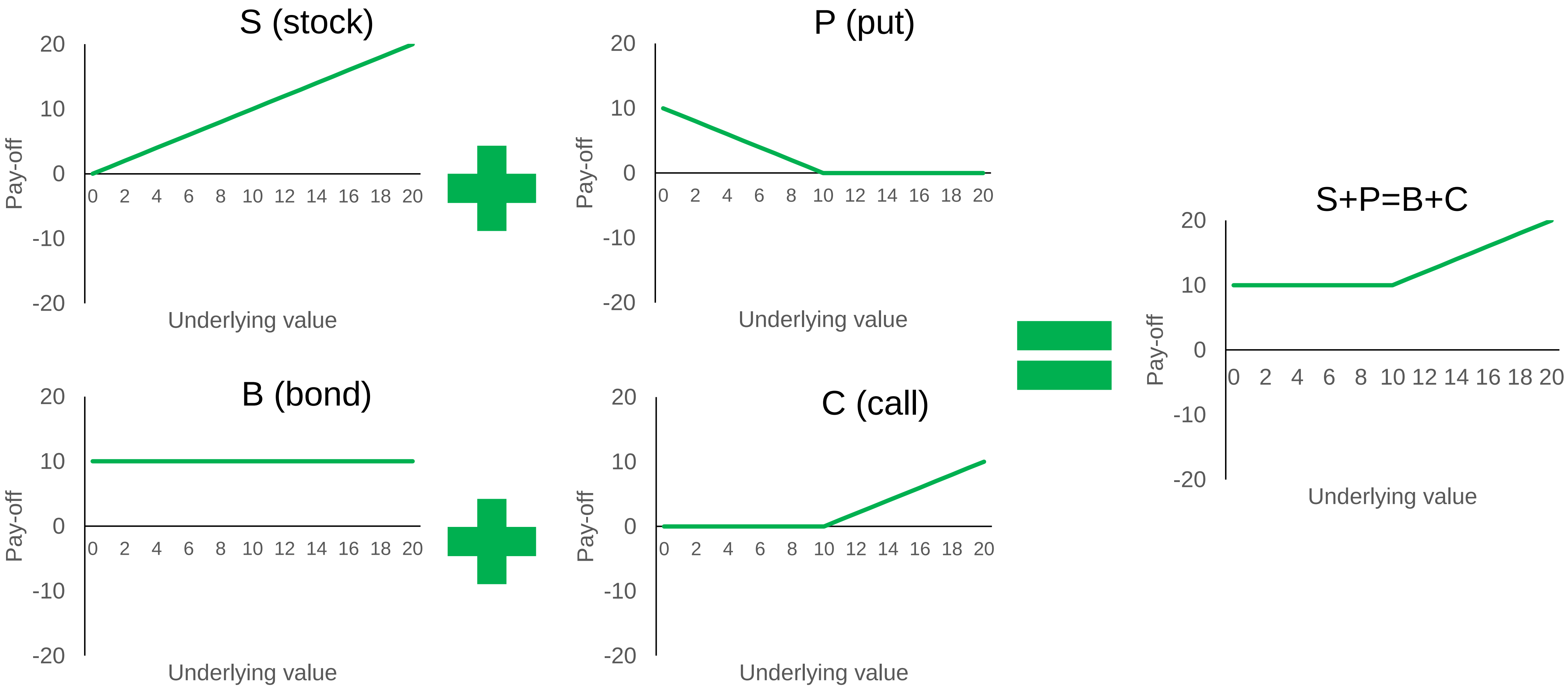
$$S + P = B + C$$

- The combined payoffs of the underlying value S and the put P are equal to the combined payoffs of a bond B and a call C ➡ **next slide shows put-call parity**





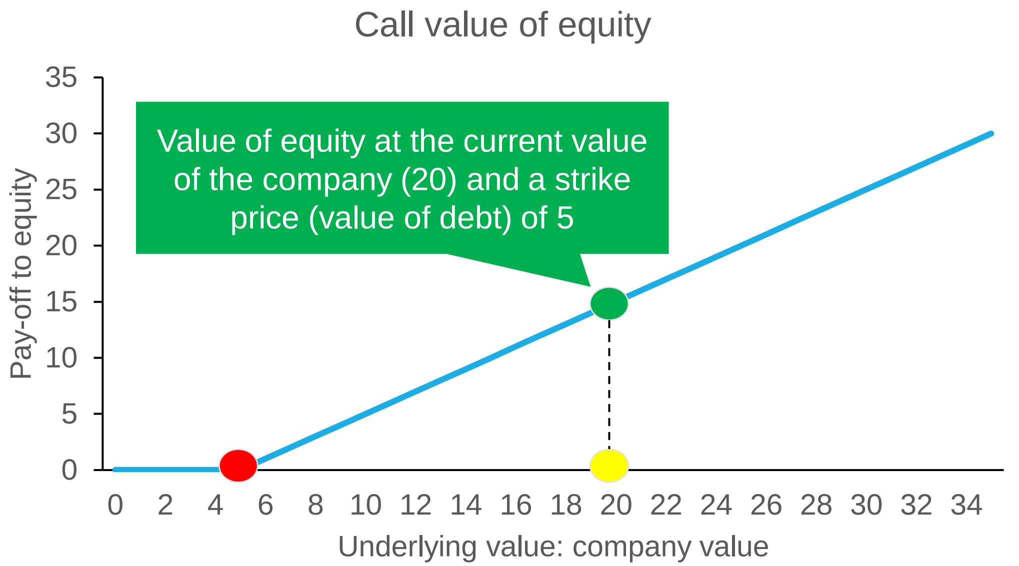
Put-call parity expressed in payoff structures



Capital structure expressed in options

- One can view capital structure in terms of options:
  - Equity can be seen as a call option on the company's assets
  - Corporate debt is effectively riskless debt minus a put on the company's assets
- The NPV of projects is the driver of the value of assets and the value of equity
  - If the NPV of projects falls below 5, equity value goes to 0
  - For all values above 5, the value of equity moves proportionately with the NPV of projects

NPV of projects	20	Debt	5
		Equity	15
Total assets	**20**	**Total liabilities**	**20**



$$Equity\ as\ a\ call = C = \max(Assets - Debt, 0)$$

Corporate debt in terms of options

- Since equity can be expressed in options, it follows from put-call parity that corporate debt can also be expressed in terms of options

$$S + P = B + C$$

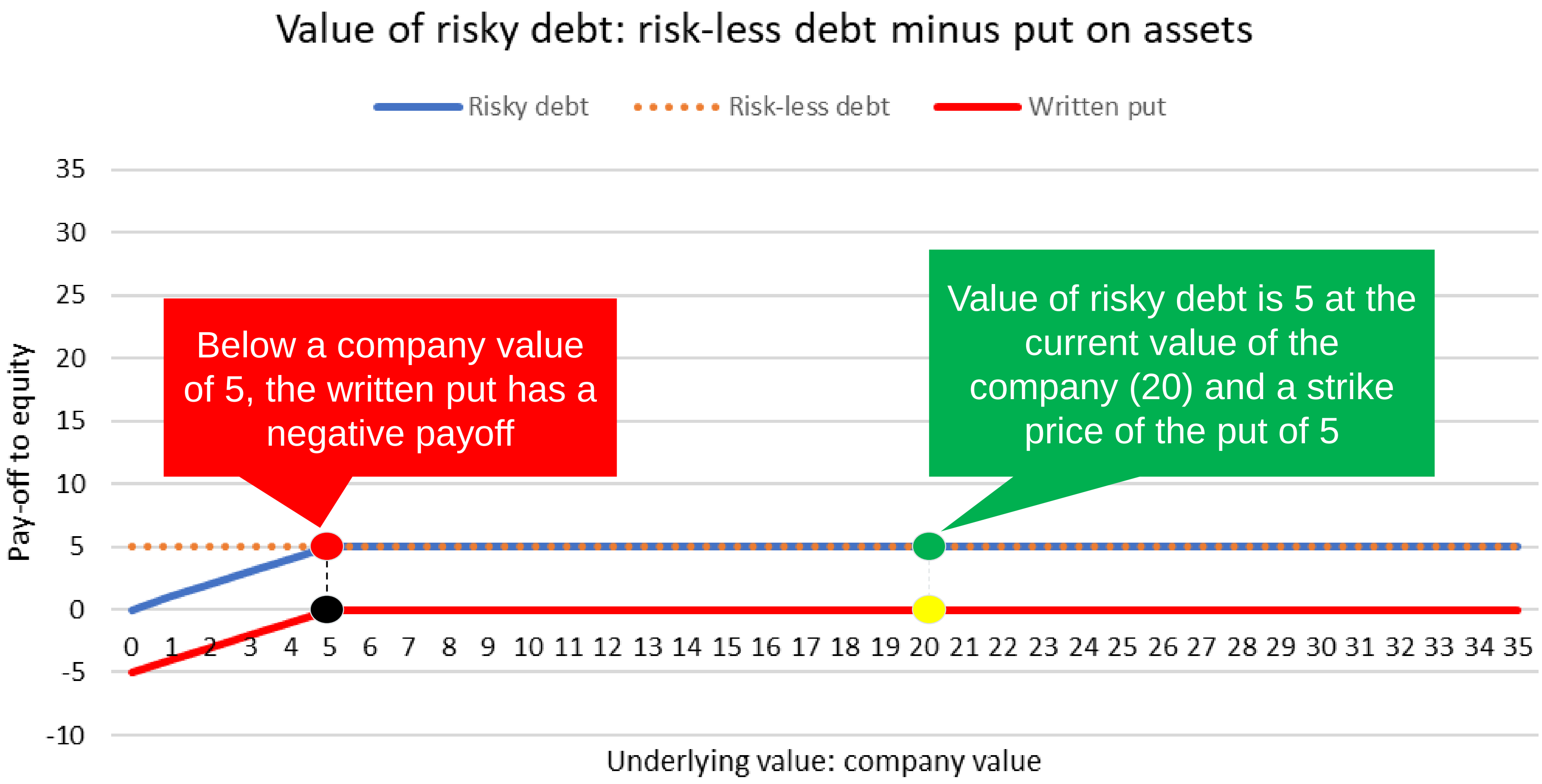
Where S = assets, P = put on assets, B = riskless debt and C = call on assets

- As such: assets = $S = B - P + C$
- Assets are financed by risky debt and equity, and since equity $= C$, it follows:

$$Risky\ debt = Assets - Equity = (B - P + C) - C = B - P$$

Risky debt = riskless debt – put on assets

Corporate debt in terms of options



Option quotations

- Options are most commonly traded on stocks
  - Example of 3M stock with 3 strike prices

Strike*	C or P	Bid*	Ask*	Open interest
60	C	53.50	55.55	0
	P	2.97	3.90	20
110	C	20.50	21.75	17
	P	16.90	18.00	64
160	C	5.65	6.45	25
	P	49.15	50.70	2

* Per October 2022, with expiration in January 2025

- **Strike price** is the fixed price at which the owner of the option can by or sell the underlying security
- **Bid price** is the price at which the market-makers are willing to buy the option
- **Ask price** is the price at which the market-makers are willing to sell the option
- **Open interest** refers to the number of option contracts that are held by traders in active positions

Option quotations

- As of October 2022, 3M stock was $113.45

Strike*	C or P	Bid*	Ask*	Open interest
60	C	53.50	55.55	0
	P	2.97	3.90	20
110	C	20.50	21.75	17
	P	16.90	18.00	64
160	C	5.65	6.45	25
	P	49.15	50.70	2

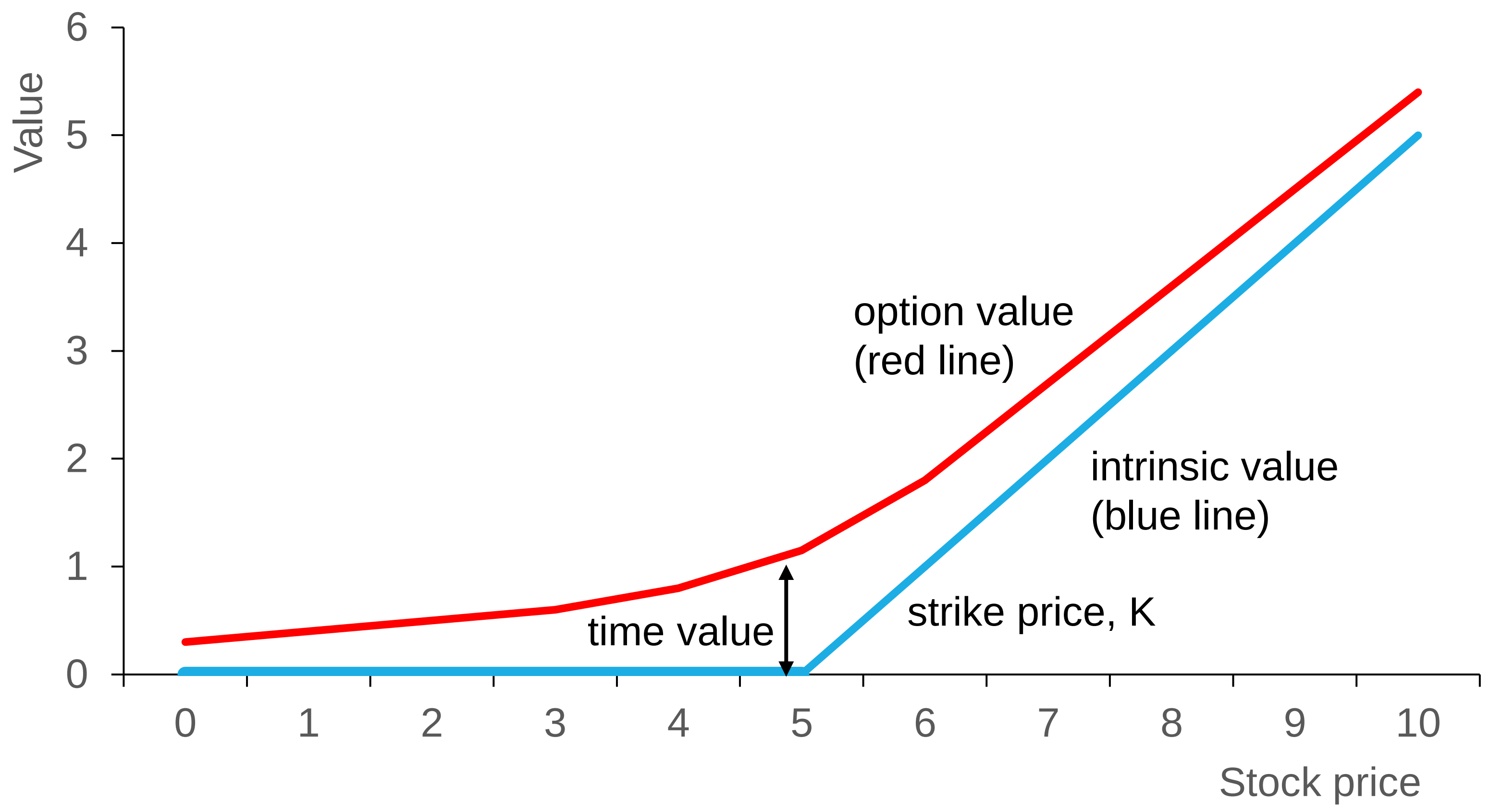
* Per October 2022, with expiration in January 2025

$$\boldsymbol{Option\ value = intrinsic\ value + time\ value}$$

(see next slide)

- At $60, the call option is very much *'in-the-money'*, whereas the put option is *'out-of-the-money'*
- The difference between the stock price and exercise price is the *intrinsic value of the option*
  - For $60 call option: $113.45 - $60 = $53.45
  - For $60 put option: $60 - $113.45 = 0
- The average of the bid and ask is the option value
- The option value minus the *intrinsic value* is called the *time value of the option*
  - For $60 call: ($55.55 + $53.50) / 2 = $54.53 - $53.45 = $1.08
  - For $60 put option: ($3.90 - $2.97) / 2 = $3.44 - $0 = $3.44

Option value = intrinsic value + time value



Valuing options

- The non-linear nature of option payoffs makes valuation of options difficult
- The *binomial option pricing model* prices options by making the simplistic assumption that at the end of the next period ($r_f = 3\%$), the underlying value has only two possible values
- The two-state single period model values a call option by building a replicating portfolio, which is a portfolio of other securities with the same value as the option in one period

Stock: S = $10
Bond: B = $1

up

Stock:
S_u = $12

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_u = max($12-$10,$0) = $2

down

Stock:
S_d = $8

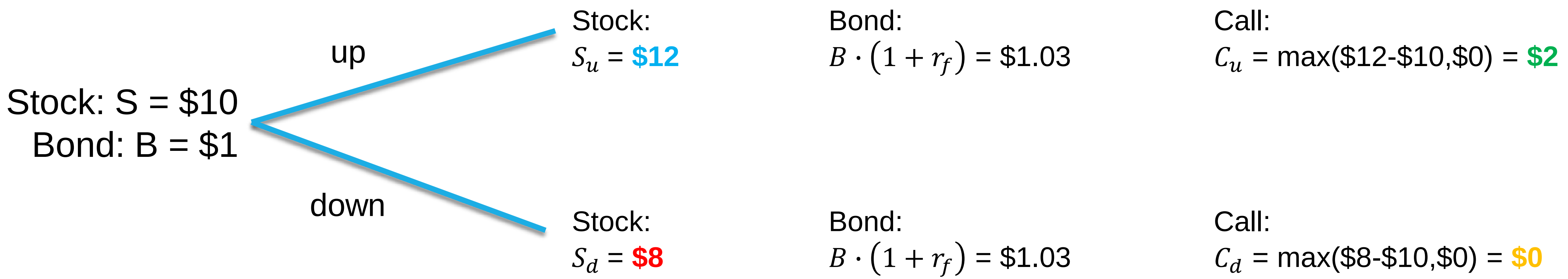
Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_d = max($8-$10,$0) = $0

Binomial model – step 1

- The idea is to buy stock in such proportions that they give the same payoffs as the call
- The first step is to determine the option delta Δ (or hedge ratio), which is the number of stocks needed to replicate or hedge the call:

$$\Delta = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}} = \frac{C_u - C_d}{S_u - S_d} = \frac{\$2 - \$0}{\$12 - \$8} = 0.5$$



Binomial model – step 2

- The second step is to determine the number of bonds needed to finance the stock position
- The replication, for one time period later, is:
  - For the up-state: $C_u = \Delta \cdot S_u - (1 + r_f) \cdot B$
  - For the down-state: $C_d = \Delta \cdot S_d - (1 + r_f) \cdot B$

To derive the number of bonds: $B = \frac{\Delta \cdot S_u - C_u}{1 + r_f}$

Up-state: $B = \frac{0.5 \cdot \$12 - \$2}{1.03} = \$3.88$

Down-state: $B = \frac{0.5 \cdot \$8 - \$0}{1.03} = \$3.88$

Stock: S = $10
Bond: B = $1

up

Stock:
S_u = **$12**

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_u = max($12-$10,$0) = **$2**

down

Stock:
S_d = **$8**

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_d = max($8-$10,$0) = **$0**

Binomial model – step 3

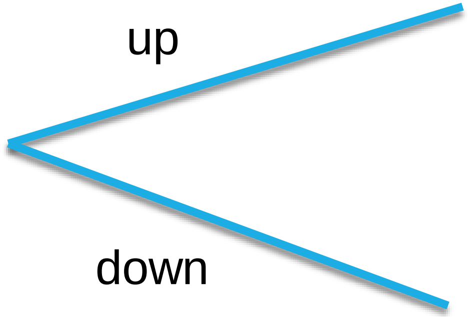
- The final step is to determine the price of the call
- The price of the call option in the binomial model is as follows:

Value of call = [delta x stock price] - [bonds]

$$C = \Delta \cdot S - B$$

$$C = 0.5 \cdot \$10 - \$3.88 = \$1.12$$

Stock: S = $10
Bond: B = $1



Stock:
S_u = $12

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_u = max($12-$10,$0) = $2

Stock:
S_d = $8

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_d = max($8-$10,$0) = $0

Binomial model – step 4 (overview)

Instrument	Period t = 0	Period t = 1 Up	 Down
Replicating portfolio	$\Delta \cdot S - B =$ 0.5*$10 - $3.8835 = $1.1165	$\Delta \cdot S_u - (1 + r_f) \cdot B =$ 0.5 * $12 – (1.03) * $3.8835 = $6 - $4 = $2	$\Delta \cdot S_d - (1 + r_f) \cdot B =$ 0.5 * $8 – (1.03) * $3.8835 = $4 - $4 = $0
Call	$C = \$1.12$	$C = \$2$	$C = \$0$

Stock: S = $10
Bond: B = $1

up

Stock:
S_u = $12

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_u = max($12-$10,$0) = $2

down

Stock:
S_d = $8

Bond:
$B \cdot (1 + r_f)$ = $1.03

Call:
C_d = max($8-$10,$0) = $0

Put option in binomial model

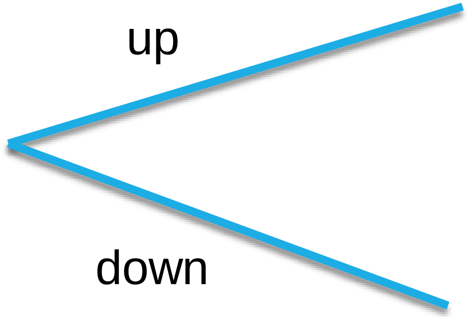
- The calculation of the put price occurs in a similar way to the call price

Option delta $\Delta = \frac{\text{spread of possible option prices}}{\text{spread of possible stock prices}} = \frac{P_u - P_d}{S_u - S_d} = \frac{\$0 - \$2}{\$12 - \$8} = -0.5$

Number of bonds $B = \frac{-\Delta \cdot S_u + P_u}{1 + r_f} = \frac{-(-0.5) \cdot \$12 + \$0}{1.03} = \$5.83$

Value of put = [delta x stock price] + [bonds] = $P = \Delta \cdot S + B = -0.5 \cdot \$10 + \$5.83 = \$0.83$

Stock: S = \$10
Bond: B = \$1

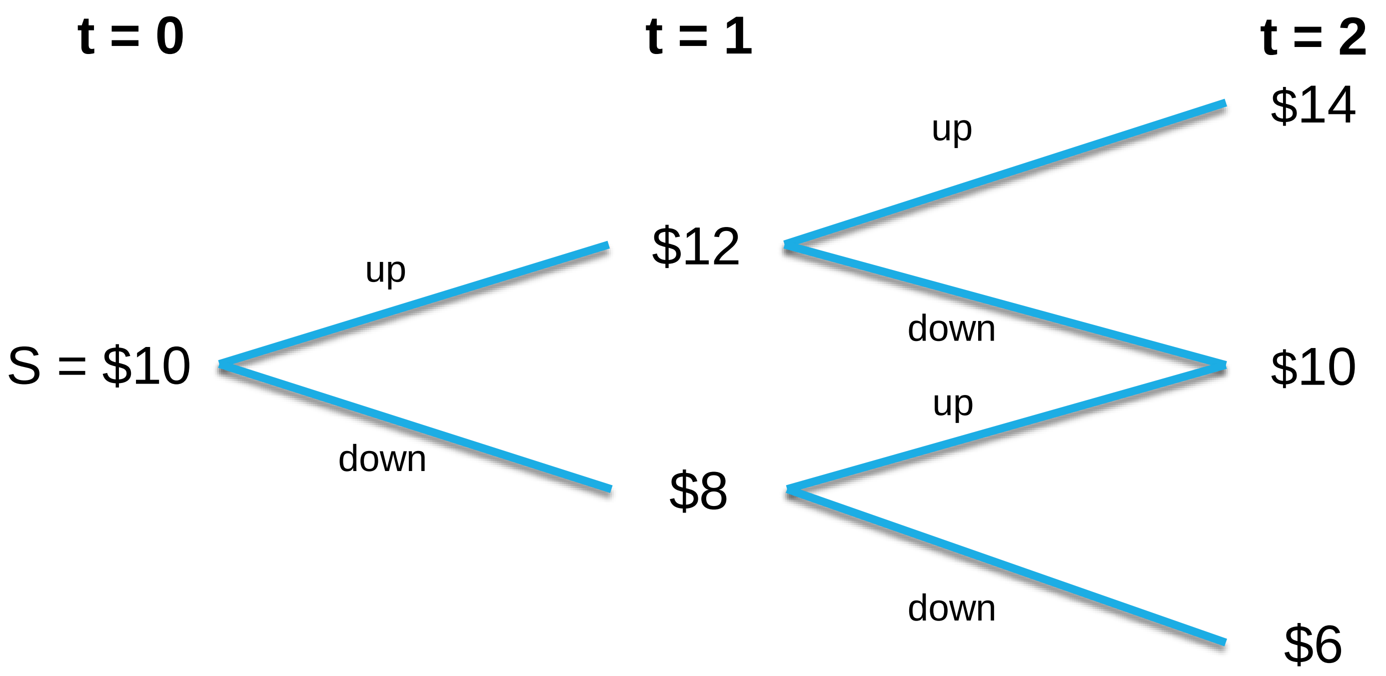


Up state:
Stock: S_u = \$12
Bond: $B \cdot (1 + r_f)$ = \$1.03
Call: P_u = max(\$10-\$12,\$0) = \$0

Down state:
Stock: S_d = \$8
Bond: $B \cdot (1 + r_f)$ = \$1.03
Call: P_d = max(\$10-\$8,\$0) = \$2

Multiperiod binomial model

- The formula for the option delta Δ can be interpreted as the sensitivity of the option's value to changes in the stock price at each point in time
- We can value the call option in the multiperiod binomial tree **by working backwards from t = 2**



Option deltas

$$\Delta = \frac{C_u - C_d}{S_u - S_d} = \frac{\$4 - \$0}{\$14 - \$10} = 1$$

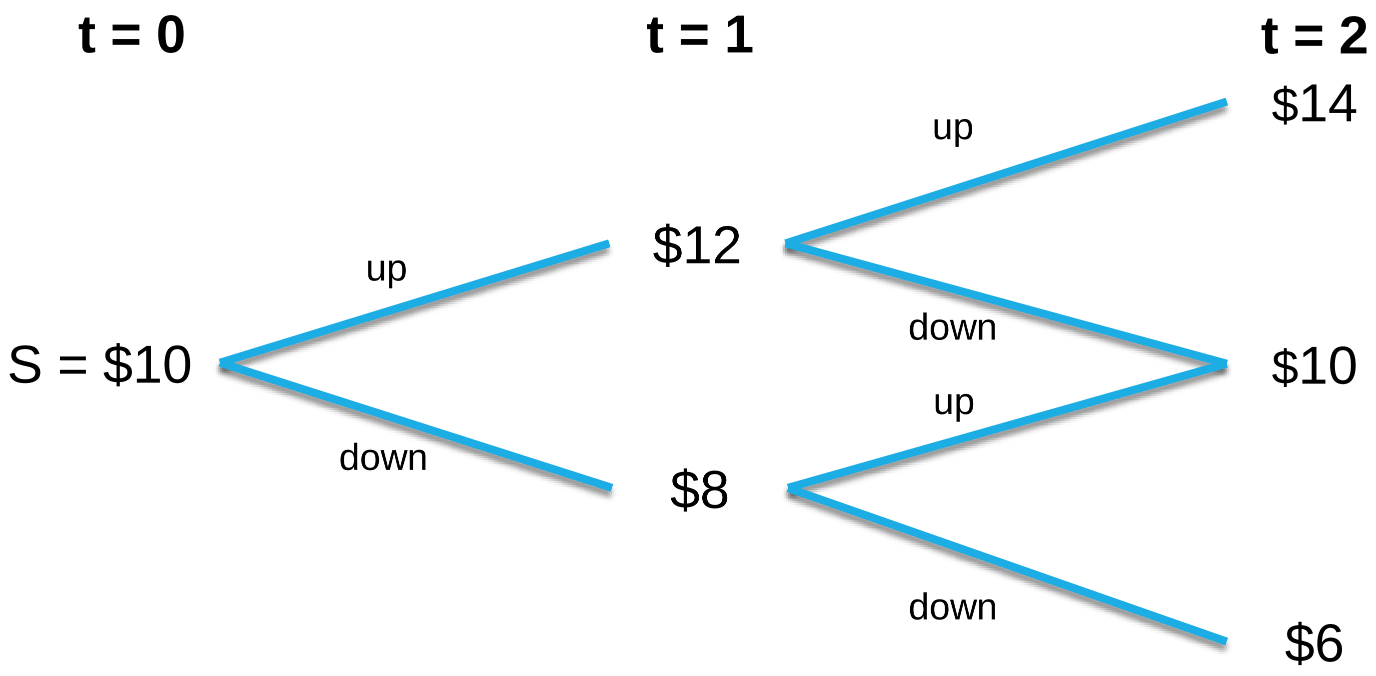
$$\Delta = \frac{C_u - C_d}{S_u - S_d} = \frac{\$0 - \$0}{\$10 - \$6} = 0$$

A call with zero payoffs in the future, also has zero value today

Multiperiod binomial model

$$B = \frac{\Delta \cdot S_u - C_u}{1 + r_f} = \frac{1 \cdot \$14 - \$4}{1.03} = \$9.709$$

$$C = \Delta \cdot S - B = 1 \cdot \$12 - \$9.709 = \$2.291$$ ← Value of call option in the up-state at t = 1



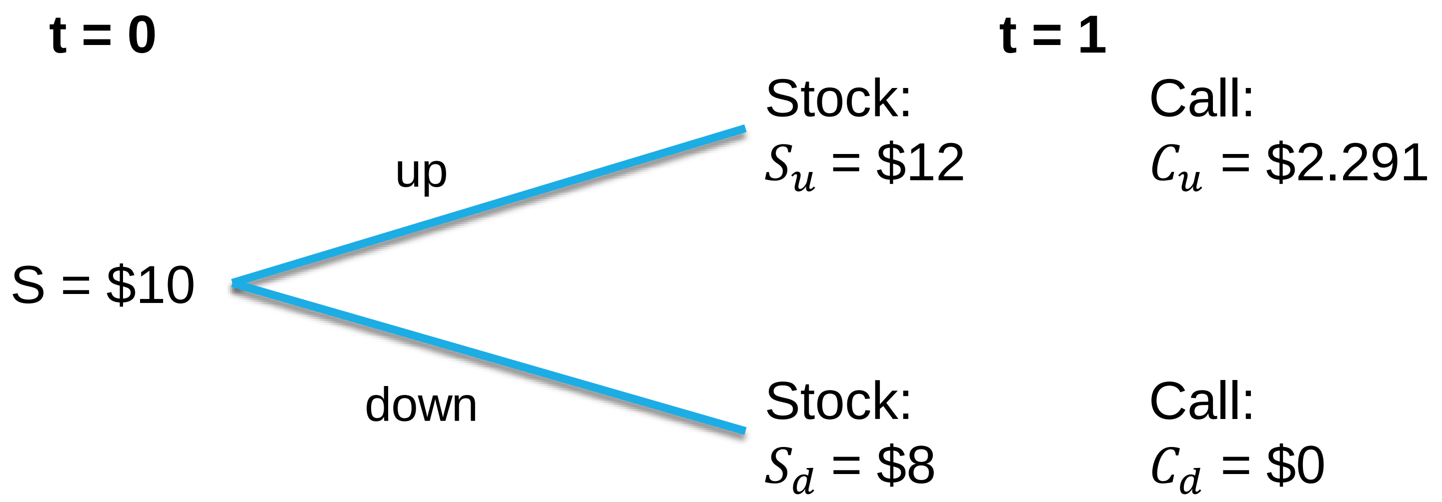
Multiperiod binomial model

$$\Delta = \frac{C_u - C_d}{S_u - S_d} = \frac{\$2.291 - \$0}{\$12 - \$8} = 0.573$$

$$B = \frac{\Delta \cdot S_u - C_u}{1 + r_f} = \frac{0.573 \cdot \$12 - \$2.291}{1.03} = \$4.449$$

$$C = \Delta \cdot S - B = 0.573 \cdot \$10 - \$4.449 = \$1.279$$

Value of call option at t = 0,
Slightly higher than the one-period call with a value of $1.12

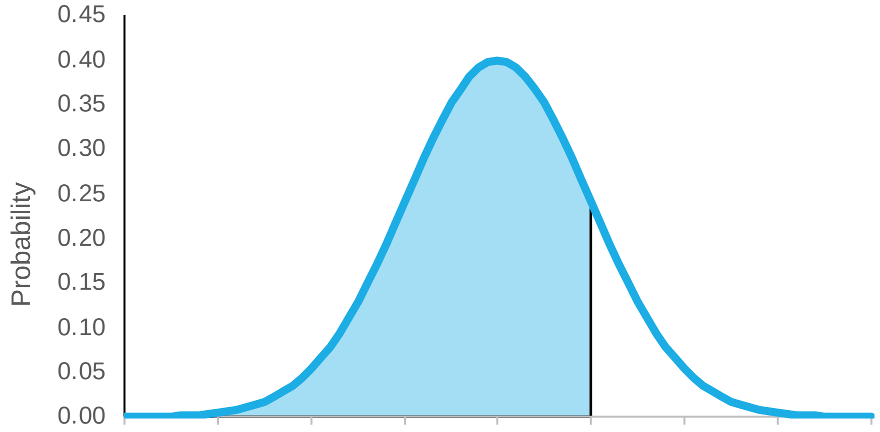


Option pricing models

1. From binomial option pricing model for two periods

2. To multiperiod binomial option pricing model for multiple periods
   - Refine the tree by cutting the maturity of call option in ever smaller periods
   - Ending up in a continuous returns distribution

3. To Black-Scholes option pricing model
   - Based on normal distribution of returns



Black-Scholes option pricing model

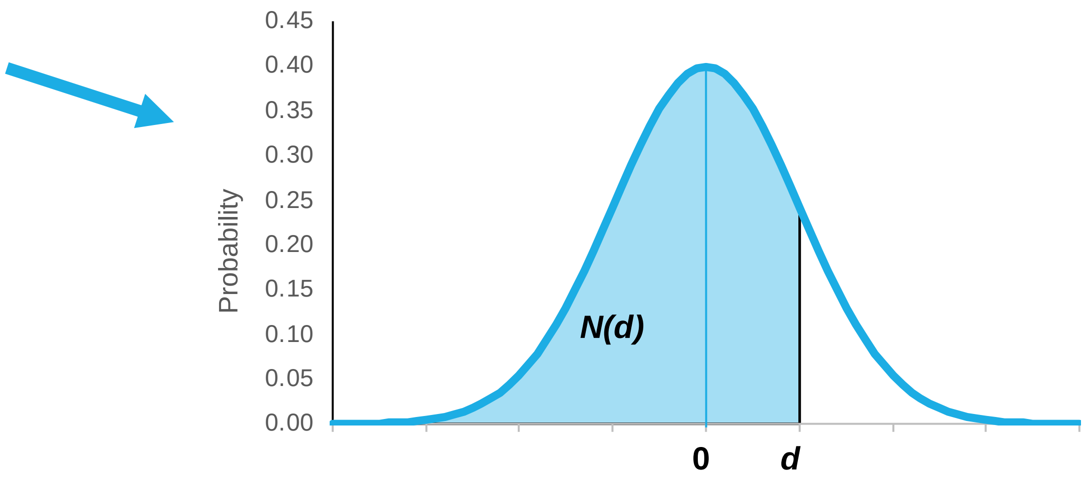
- The Black-Scholes formula for the price of a call on a non-dividend paying stock follows the set-up of the binominal model:

Value of call = [delta x stock price] – [bonds]

$$C = N(d_1) \cdot S - N(d_2) \cdot PV(K)$$

Where:

- S is the current price of the underlying stock
- $PV(K)$ is the present value of a risk-free zero-coupon bond that pays K
- K is the exercise price
- $N(d)$ is the cumulative normal probability distribution
- $d_1 = \frac{\ln[S/PV(K)]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$
- $d_2 = d_1 - \sigma\sqrt{T}$
- σ is annual volatility of the stock's returns
- T refers to the number of years until expiration



European dividend paying stocks

- The Black-Scholes formulas are derived for non-dividend paying stocks
  - They can easily be adjusted for dividend paying stocks

- The European call option holder has no rights to any dividends paid out prior to expiration
  - European options can only be exercised at the expiration date
  - American options allow the owner to exercise the option at any time

- For European call options, we can deduct the present value of missed dividends $PV(Div)$ from the stock price S:

$$S^* = S - PV(Div)$$

Implied volatility & risk of options

- While most parameters are easy to calculate, volatility of stock price σ is more difficult to calculate
  - The most direct way is to calculate a stock's volatility from historical stock prices
  - Traders sometimes take a shortcut by deriving a stock's volatility from current market prices of traded options
  - Estimating a stock's volatility that is implied by an option's market price is called implied volatility

- We can derive the risk of an option from the underlying replicating portfolio
  - The beta of the option β_{option} is a weighted average of the beta of the stock and the bond

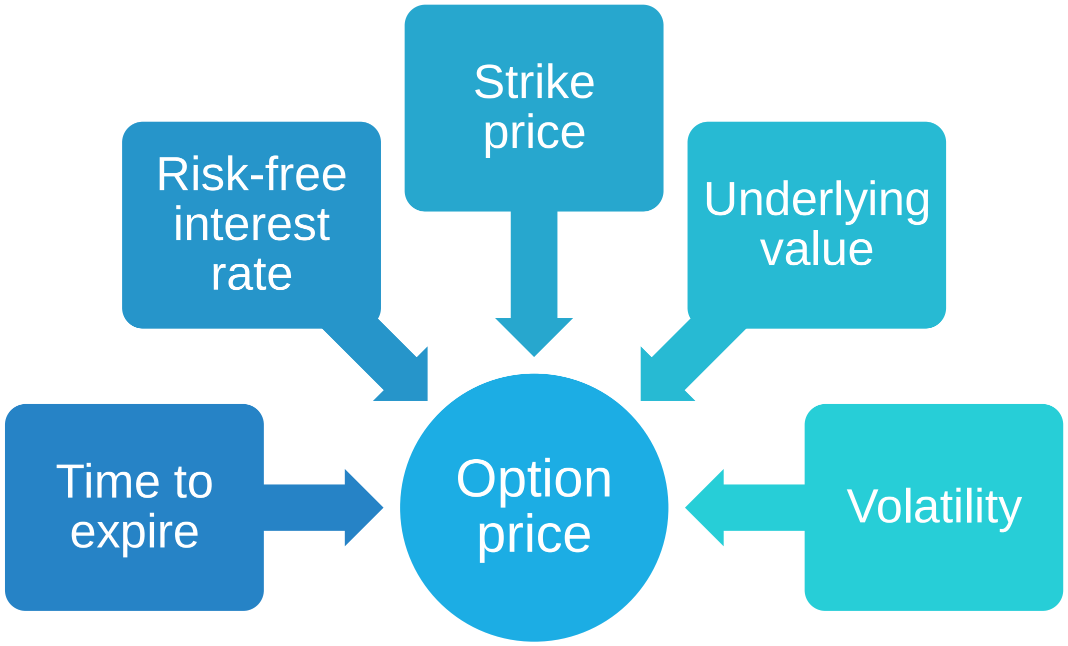
$$\beta_{option} = \frac{\Delta \cdot S}{\Delta \cdot S + B} \cdot \beta_{stock} + \frac{B}{\Delta \cdot S + B} \cdot \beta_{bond}$$

  - Given that the bond is risk-less, $\beta_{bond} = 0$ → $\beta_{option} = \frac{\Delta \cdot S}{\Delta \cdot S + B} \cdot \beta_{stock}$
  - While the beta of a call is positive, the beta of a put is typically negative reflecting the short position in stocks

Drivers of option prices

- The Black-Scholes formulas reveal the drivers of option prices
- Drivers have different signs for calls and puts (see table)

Driver	Call option	Put option
Underlying value	+	-
Volatility	+	+
Strike price	-	+
Time to expiration	+	+
Risk-free interest rate	+	-



Real options

- A real option is the opportunity to make a particular business decision → gives flexibility
- In contrast to financial options:
  - They are not exchange traded
  - There is no formal option contract
  - There is no clear counterparty
- Similar to financial options, real options have a payoff that depends on factors such as the underlying value and a strike price on that underlying value
  - Being long in real options provides valuable flexibility to exercise an opportunity
  - Being short in real options can be very risky and value destructive
- You typically cannot price real options by no-arbitrage principles since these real options are not redundant
  - This means you cannot make a replicating portfolio to price real options

Application of real options

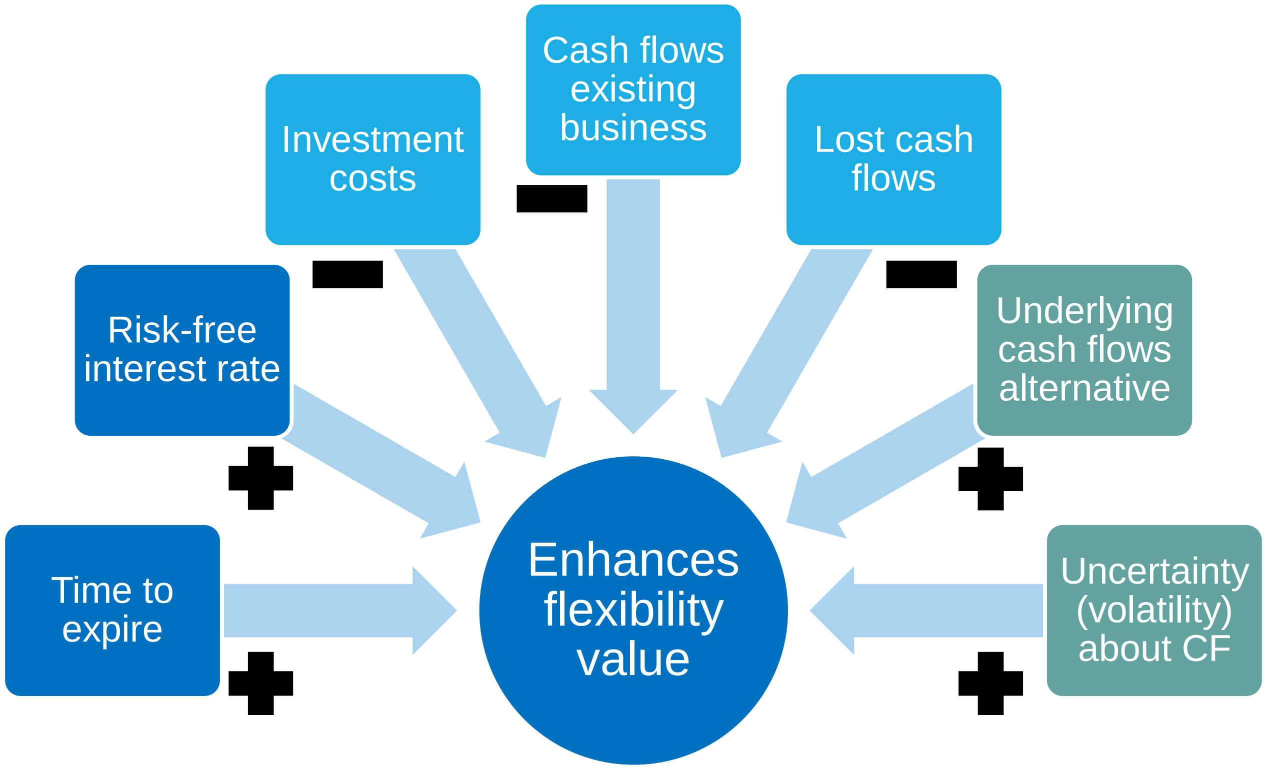
- Many corporate assets, particularly growth opportunities, can be viewed as call options (Myers, 1977)
- Such ‘real options’ depends on discretionary future investment by the firm
- Example: a company has developed an innovative technology
  - The company now has a put option on the production costs of the innovative technology
  - The strike price is the production cost of the traditional technology
  - When costs of the innovative technology go down, the put option comes ‘in-the-money’
- Luehrman (1998): “In financial terms, a business strategy is much more like a series of options than a series of static cash flows”

Types of real options

- Koller, Goedhart and Wessels (2020) provide a classification of real options:
  - **Option to defer** (a call): flexibility to wait and do the same action later
  - **Abandonment option** (put): option to stop an operation
  - **Follow-on (compound) option**: a series of options on options
  - **Option to expand** (call) **or contract** (put): flexibility in the size of the operations
  - **Option to extend** (call) **or shorten** (put): flexibility to adapt the lifetime of an operation
  - **Option to increase scope** (call): flexibility to add other operations
  - **Switching options**: flexibility to choose between different operations

Drivers of real options

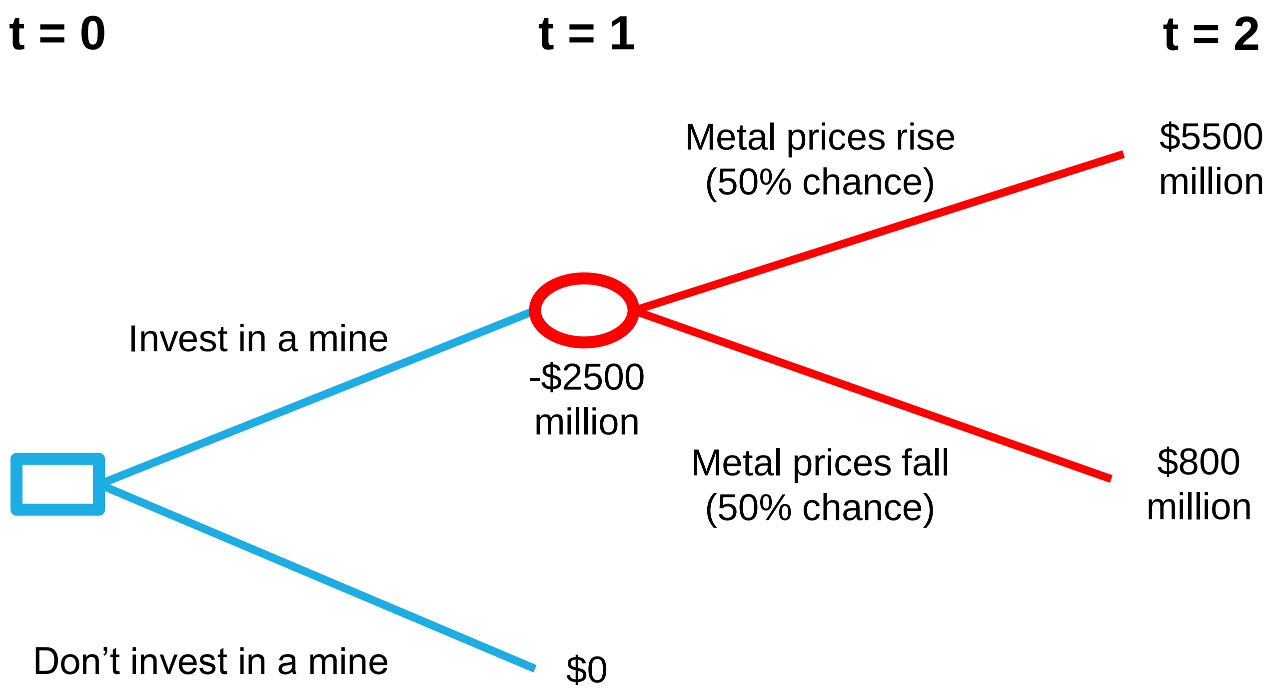
- Drivers of real options follow from the drivers of financial options
  - Key point: uncertainty - measured as volatility of cash flows - enhances the flexibility value



Source: Adapted from Koller, Goodhart and Wessels (2020)

Decision tree analysis for real options

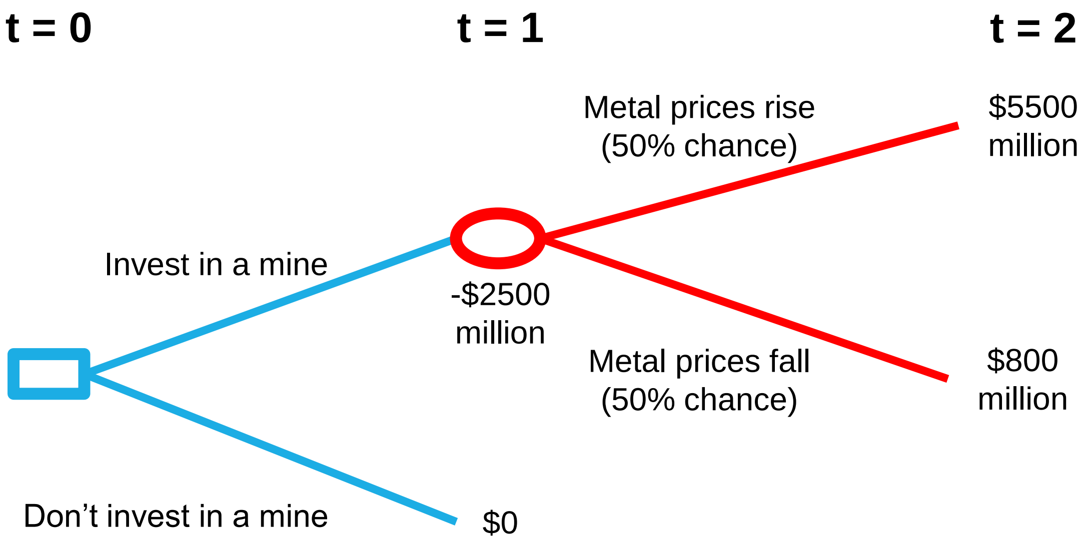
- Real options can be analysed using decision trees, which are a graphical representation of future decisions under uncertainty



Example: investing in a mine

- The investment at t = 0 effectively buys the company an exposure to metal prices
- Once the mine has been built at t = 0, the investment in the mine is not a call anymore at t = 1
- The call lies before that at t = 0: the choice to build the mine or not

Decision tree analysis for real options



Conclusion: the expected payoff is positive, so the company should invest in the mine

Scenario	Payoff (1)	Probability (2)	Expected payoff (3)	Previous payoff (4)	Total payoff (5)	Total expected payoff (6)
Calculation			(1)*(2)		(1)+(4)	(5)*(2)
Metal prices rise	5,500	50%	2,750	-2,500	3,000	1,500
Metal prices fall	800	50%	400	-2,500	-1,700	-850
Total		100%	3,150			**650**

Corporate use of real options

- In corporate practice, the use of real options is not as widespread as academics had imagined
  - Managers tend to favour DCF analysis in capex decisions, or simpler but flawed alternatives, such as the payback criterion

- Triantis and Borison (2001) identify three main corporate uses of real options:
  - As a strategic way of thinking
  - As an analytical valuation tool
  - As an organisation-wide process for evaluating, monitoring, and managing capital investments

- Often, managers are not aware of the options they have, since these options are not explicitly presented as such (‘opaque framing’) and are thus not identified in the first place

Real call positions driven by E and S

- How to think about real call options driven by E and S?
  - On the long side, calls result from grasping E and S opportunities
  - On the short side are incumbent companies that are currently destroying value on E or S
- The intrinsic value of the long call increases with the size of the positive externality (opportunity)
- The value of the short call decreases with the size of the negative externality (incumbent technology)
- The intrinsic value of the real call option increases with the attractiveness of the new technology and decreases with the attractiveness of the incumbent technology
- As the new technology becomes competitive, the value of the long call goes up and might come in-the-money, while that of the short call falls

E and S drivers of real call options on F

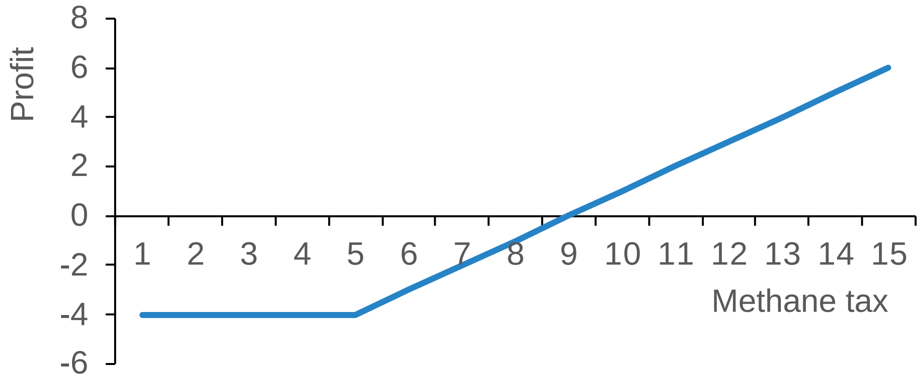
		Long call	Short call
Underlying value	Size of the positive externality of the new technology relative to the old technology	+	-
	CFs new technology	+	-
Strike price	CFs incumbent technology	-	+
Volatility	Transition tensions	+	-
Time to expiration	Economic life of the incumbent technology	+	-
Risk-free interest rate	PV of the incumbent technology (strike price)	+	-

Example of DSM's product Bovaer

- DSM has developed the product Bovaer, which reduces methane emissions of cows
  - The development cost of Bovaer is the option premium at €4 per unit product
  - The strike price of the call is the production cost of Bovaer at €5 per unit product

    Note: these numbers are fictional and intended for illustrative purposes

- DSM will start producing and selling Bovaer (i.e. exercising the call option) as soon as the methane tax exceeds the strike price
  - DSM will break even at a methane tax of €9



- From a short call perspective: as the transition tensions and/or size of externality increases (i.e. the methane tax rises), the risk of doing nothing increases
  - This should push companies to consider strategies that avoid this risk by phasing out old technologies

Real put positions driven by E and S

- Practical example of a short put: Boeing taking short-curs in safety
  - Every year Boeing benefits from cost reductions (put premium)
  - Until one year this led to an accident and massive costs

- The underlying value is the safety level of the aircraft: the probability that no accidents happen which will result in large fines

- Safe transport and arrival of passengers is the threshold and strike price: below that price, costs rise significantly

		Long put	Short put
Underlying value	Safety level as measured by: - Health of passengers - Avoiding multi-billion dollar fines	 - -	 + +
Strike price	Passengers arrive safely due to safety investment	+	-
Volatility	Swings in resulting safety levels	+	-
Time to expiration	Use time of the planes produced	+	-
Risk-free interest rate	PV of the safety investment	-	+

Comparing call and put examples

- There are at least three differences between the put and call examples
  - The put examples are not a transition risk: it is not a bigger societal challenge, but purely the result of decisions to economise on safety
  - The short puts are not driven directly by competing products: there is no new technology that can eradicate the negative externality
  - There is no clear counterparty that has the exact mirror position

- There are also similarities:
  - Both long calls and short puts have a competitive element:
    - Companies that invest in long calls increase their competitive composition by frontloading new technology
    - Companies that write short puts increase their competitive position by cutting costs

Integrated value as a set of real options

- Companies can create options for specific stakeholders (i.e. shareholders, employees) at the expense of other stakeholders
- The integrated value perspective helps to make such situations visible, by explicitly comparing EV, FV and SV
- We can express the market value balance sheet as a combination of risk-free assets and liabilities and options on F, S, and E

S assets	20	S debt	5
		S equity	15
E assets	15	E debt	25
		E equity	-10
F assets	25	F debt	5
		F equity	20
Total integrated assets	**60**	**Total integrated liabilities**	**60**

$$Assets = Risky\ debt + Equity = B - P + C$$

$$IV = (E\ bond + E\ call - E\ put) + (F\ bond + F\ call - F\ put) + (S\ bond + S\ call - S\ put)$$

see next slide

IV balance sheet

EV =	E bond	+ call on E	- put on E
example:	+ : natural capital improvements realised - : natural capital destruction caused	still to be realised emission savings	biodiversity damage resulting from activities
+			
FV =	**F bond**	**+ call on F**	**- put on F**
example:	CF from business as usual	potential additional CF from current / new projects / products	potential reductions in CF from current / new projects / products
+			
SV =	**S bond**	**+ call on S**	**- put on S**
example:	+ : health improvements realised - : health reductions caused	still to be realised health improvements	still to be experienced health reductions from, for example, savings on safety
=			
IV =	**I bond**	**+ call on I**	**- put on I**

Conclusions

- Financial options are contracts that give the owner the right to buy or sell a security at a pre-specified price
- The seller or writer, who is short the option, has the opposite position of the buyer, and has to exercise the contract if the buyer wants to do so
- A real option is the opportunity to make a particular business decision, exemplifying the value of flexibility
- One can visualise both financial options and real options with decision trees and payoff graphs
- Real options on F can have E or S drivers: payoff in terms of F, but with E or S as the underlying values
- Companies have a lot of put options against society, but awareness of it is low: this calls for an integrated view on options or integrated value expressed in real options