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e-companion
ONLY AVAILABLE IN ELECTRONIC FORM

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e-Companion

EC.1. Lemma 1

Lemma 1: *The optimal bets of agent i in P3 while classifying t is:* $q^*_{itj} = p_{itj}(w_{it}+m)\ \forall j \in J$.

Proof Lemma 1

$Z_3 = \max_{q_{itj}}\ p_{it1}ln(q_{it1}O_{t1}) + p_{it2}ln(q_{it2}O_{t2})$ (26)

S.T. $q_{it1} + q_{it2} = w_{it}+m$ (27)

$q_{itj} \geq 0$ (28)

Using Lagrangian multipliers λ_1, λ_2, and λ_3, we get

$L_3 = p_{it1}ln(q_{it1}O_{t1}) + p_{it2}ln(q_{it2}O_{t2}) + \lambda_1(w_{it} + m - q_{it1} - q_{it2}) + \lambda_2(q_{it1} - 0) + \lambda_3(q_{it2} - 0)$ (29)

$\partial L_3/\partial q_{it1} = 0 \Rightarrow p_{it1}O_{t1}/q_{it1}O_{t1} - \lambda_1 = 0$ (30)

$\partial L_3/\partial q_{it2} = 0 \Rightarrow p_{it2}O_{t2}/q_{it2}O_{t2} - \lambda_1 = 0$ (31)

$\partial L_3/\partial \lambda_1 = 0 \Rightarrow w_{it} + m - q_{it1} - q_{it2} = 0$ (32)

$\lambda_2(q_{it1} - 0) = 0 \Rightarrow \lambda_2 q_{it1} = 0$ (33)

$\lambda_3(q_{it2} - 0) = 0 \Rightarrow \lambda_3 q_{it2} = 0$ (34)

simplify (30) and (31)

$p_{it1}/q_{it1} - \lambda_1 = 0$ (35)

$p_{it2}/q_{it2} - \lambda_1 = 0$ (36)

combine (35) and (36)

$p_{it1}/q_{it1} = p_{it2}/q_{it2}$ (37)

combine (32) and (37)

$p_{it1} / q_{it1} = p_{it2} / (w_{it} + m - q_{it1})$ (38)

$p_{it2} / q_{it2} = p_{it1} / (w_{it} + m - q_{it2})$ (39)

simplify (38) and (39) (note that $p_{it1} = 1 - p_{it2}$)

$p_{it1} (w_{it} + m) - p_{it1} q_{it1} = q_{it1} - p_{it1} q_{it1}$ (40)

$p_{it2} (w_{it} + m) - p_{it2} q_{it2} = q_{it2} - p_{it2} q_{it2}$ (41)

simplify (40) and (41)

$q_{it1} = p_{it1} (w_{it} + m)$ (42)

$q_{it2} = p_{it2}(w_{it} + m)$ (43)

Use the Hessian matrix for $p_{it1}ln(q_{it1}O_{t1}) + p_{it2}ln(q_{it2}O_{t2}) + \lambda_1(w_{it} + m - q_{it1} - q_{it2}) + \lambda_2(q_{it1} - 0) + \lambda_3(q_{it2} - 0)$ to verify that L_3 has a relative maximum at the critical point obtained in (42) and (43):

$$\begin{bmatrix} \partial^2 L_3 / q_{it1}^2 & \partial^2 L_3 /(q_{it1}, q_{it2}) \\ \partial^2 L_3 /(q_{it2}, q_{it1}) & \partial^2 L_3 / q_{it2}^2 \end{bmatrix}, \text{ where} \quad (44)$$

$\partial^2 L_3 / q_{it1}^2 = -p_{it1}/q_{it1}^2$ (45)

$\partial^2 L_3 / q_{it2}^2 = -p_{it2}/q_{it2}^2$ (46)

$\partial^2 L_3 /(q_{it1}, q_{it2}) = 0$ (47)

$\partial^2 L_3 /(q_{it2}, q_{it1}) = 0$ (48)

The determinant of (44) is:
$D_3 = (\partial^2 L_3 / q_{it1}^2)(\partial^2 L_3 / q_{it2}^2) - (\partial^2 L_3 /(q_{it1}, q_{it2}))(\partial^2 L_3 /(q_{it2}, q_{it1}))$
$D_3 = (-p_{it1}/q_{it1}^2)\ (-p_{it2}/q_{it2}^2)$ (49)

Simplify (49)
$D_3 = p_{it1}\ p_{it2}/q_{it1}^2 q_{it2}^2$ (50)

$\forall j \in J$, when $0<p_{itj}<1$, then $0< q_{itj} < (w_{it}+m)$ as per (42) and (43), therefore $D_3 > 0$. Further, since $\partial^2 L_3 / q_{it1}^2 < 0$ and $\partial^2 L_3 / q_{it2}^2 < 0$ (see (45), (46)), therefore the critical point is a relative maximum. When $p_{itj} = 0$ or 1, then $D_3=0$, i.e., the Hessian is indeterminate. It can be seen from (27), (34) and (42) that when $p_{it1} = 0$, then $q_{it1}=0$, $q_{it2} = (w_{it}+m)$ and $\lambda_3 = 0$. It can similarly be verified that when $p_{it2} = 0$, then $q_{it2}=0$, $q_{it1} = (w_{it}+m)$ and $\lambda_2 = 0$.

EC.2. Lemma 2

Lemma 2: *The optimal bets of agent i in P4 while classifying t is:*

Solution a: $q^*_{it1} = p_{it1}\, km + a_{it}\dfrac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}}$ *and* $q^*_{it2} = p_{it2}\, km + a_{it}\dfrac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}}$ *, when*

$0 < p_{it1}\, km + a_{it}\dfrac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}} < km$, *and* $0 < p_{it2}\, km + a_{it}\dfrac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}} < km$;

Solution b: $q^*_{it1} = km$, *and* $q^*_{it2} = 0$, *when*

$km \le p_{it1}\, km + a_{it}\dfrac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}}$ *; and*

Solution c: $q^*_{it2} = km$, *and* $q^*_{it1} = 0$, *when*

$km \le p_{it2}\, km + a_{it}\dfrac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}}$

Proof Lemma 2

$Z_4 = \max_{q_{itj}}\ p_{it1}ln(q_{it1}O_{t1} + a_{it}) + p_{it2}ln(q_{it2}O_{t2} + a_{it})$ (51)
S.T. $q_{it1} + q_{it2} = km$ (52)
$q_{itj} \ge 0$ (53)

Using Lagrangian multipliers λ_1, λ_2, and λ_3, we get
$L_4 = p_{it1}ln(q_{it1}O_{t1} + a_{it}) + p_{it2}ln(q_{it2}O_{t2} + a_{it}) + \lambda_1(km - q_{it1} - q_{it2}) + \lambda_2(q_{it1} - 0) + \lambda_3(q_{it2} - 0)$ (54)
$\partial L_4/\partial q_{it1} = 0 \Rightarrow p_{it1}O_{t1} / (q_{it1}O_{t1} + a_{it}) - \lambda_1 + \lambda_2 = 0$ (55)
$\partial L_4/\partial q_{it2} = 0 \Rightarrow p_{it2}O_{t2} / (q_{it2}O_{t2} + a_{it}) - \lambda_1 + \lambda_3 = 0$ (56)
$\partial L_4/\partial \lambda_1 = 0 \Rightarrow km - q_{it1} - q_{it2} = 0$ (57)
$\lambda_2(q_{it1} - 0) = 0 \Rightarrow \lambda_2\, q_{it1} = 0$ (58)
$\lambda_3(q_{it2} - 0) = 0 \Rightarrow \lambda_3\, q_{it2} = 0$ (59)

given (57), (58) and (59) Z_4 has three possible solutions:
Solution a: $0 < q_{it1} < km \rightarrow 0 < q_{it2} < km \wedge \lambda_2 = 0 \wedge \lambda_3 = 0$ (60)
Solution b: $q_{it1} = 0 \rightarrow q_{it2} = km \wedge \lambda_3 = 0$ (61)
Solution c: $q_{it2} = 0 \rightarrow q_{it1} = km \wedge \lambda_2 = 0$ (62)

Agent *i* determines the optimal solution of (51) given only the constraint in (52) as given below.

To solve for Solution a combine (55), (56) and (60)
$p_{it1}O_{t1} / (q_{it1}O_{t1} + a_{it}) - \lambda_1 = p_{it2}O_{t2} / (q_{it2}O_{t2} + a_{it}) - \lambda_1$ (63)

simplify (63)
$p_{it1}q_{it2}O_{t1}O_{t2} + p_{it1}O_{t1}a_{it} = p_{it2}q_{it1}O_{t1}O_{t2} + p_{it2}O_{t2}a_{it}$ (64)

combine (57) and (64)
$kmp_{it1}O_{t1}O_{t2} - p_{it1}q_{it1}O_{t1}O_{t2} + p_{it1}O_{t1}a_{it} = p_{it2}q_{it1}O_{t1}O_{t2} + p_{it2}O_{t2}a_{it}$ (65)
$kmp_{it2}O_{t1}O_{t2} - p_{it2}q_{it2}O_{t1}O_{t2} + p_{it2}O_{t2}a_{it} = p_{it1}q_{it2}O_{t1}O_{t2} + p_{it1}O_{t1}a_{it}$ (66)

simplify (65) and (66) (note that $p_{it1} + p_{it2} = 1$)
$q_{it1}O_{t1}O_{t2} = kmp_{it1}O_{t1}O_{t2} + p_{it1}O_{t1}a_{it} - p_{it2}O_{t2}a_{it}$ (67)
$q_{it2}O_{t1}O_{t2} = kmp_{it2}O_{t1}O_{t2} + p_{it2}O_{t2}a_{it} - p_{it1}O_{t1}a_{it}$ (68)

simplify (67) and (68)
$q_{it1} = p_{it1}km + a_{it}(p_{it1}O_{t1} - p_{it2}O_{t2})/O_{t1}O_{t2}$ (69)
$q_{it2} = p_{it2}km + a_{it}(p_{it2}O_{t2} - p_{it1}O_{t1})/O_{t1}O_{t2}$ (70)

If $q_{it1} > 0$ and $q_{it2} > 0$ then Solution a is given by (69) and (70). When $q_{it1} \leq 0$, then agent i will bet as per Solution b, else when $q_{it2} \leq 0$, then agent i will bet as per Solution c.

Use the Hessian matrix for $p_{it1}ln(q_{it1}O_{t1} + a_{it}) + p_{it2}ln(q_{it2}O_{t2} + a_{it}) + \lambda_1(km - q_{it1} - q_{it2}) + \lambda_2(q_{it1} - 0) + \lambda_3(q_{it2} - 0)$ to verify that L_4 has a relative maximum at the critical point obtained in (69) and (70).

$$\begin{bmatrix} \partial^2 L_4 / q_{it1}^2 & \partial^2 L_4 /(q_{it1}, q_{it2}) \\ \partial^2 L_4 /(q_{it2}, q_{it1}) & \partial^2 L_4 / q_{it2}^2 \end{bmatrix}, \text{ where} \quad (71)$$

$$\partial^2 L_4 / q_{it1}^2 = -p_{it1}O_{t1}^2 /(q_{it1}O_{t1} + a_{it})^2 \quad (72)$$
$$\partial^2 L_4 / q_{it2}^2 = -p_{it2}O_{t2}^2 /(q_{it2}O_{t2} + a_{it})^2 \quad (73)$$
$$\partial^2 L_4 /(q_{it1}, q_{it2}) = 0 \quad (74)$$
$$\partial^2 L_4 /(q_{it2}, q_{it1}) = 0 \quad (75)$$

The determinant of (71) is:
$D_4 = (\partial^2 L_4 / q_{it1}^2)(\partial^2 L_4 / q_{it2}^2) - (\partial^2 L_4 /(q_{it1}, q_{it2}))(\partial^2 L_4 /(q_{it2}, q_{it1}))$
$D_4 = (-p_{it1}O_{t1}^2 /(q_{it1}O_{t1} + a_{it})^2)(-p_{it2}O_{t2}^2 /(q_{it2}O_{t2} + a_{it})^2)$ (76)

Simplify (76)
$D_4 = p_{it1}p_{it2}O_{t1}^2O_{t2}^2 /((q_{it1}O_{t1} + a_{it})^2(q_{it2}O_{t2} + a_{it})^2)$ (77)

$\forall j \in J$, when $0<p_{itj}<1$, then $D_4 > 0$. Note that $O_{tj} \geq 1$ by definition, $a_{it}>0$ given $w_{it} > (k-1)m$. Further since $\partial^2 L_4 / q_{it1}^2 < 0$ and $\partial^2 L_4 / q_{it2}^2 < 0$ (see (72), (73)), therefore the critical point is a relative maximum. When $p_{itj} = 0$ or 1, then $D_4=0$, i.e., the Hessian is indeterminate. It can be seen from (52) and (69) that when $p_{it1} = 0$, then $q_{it1} \leq 0$, $q_{it2} \geq km$, i.e., $p_{it2}\, km + a_{it}\dfrac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}} \geq km$, as per (70). It can similarly be verified that when $p_{it2} = 0$, then $q_{it2} \leq 0$, $q_{it1} \geq km$, i.e., $p_{it1}\, km + a_{it} \frac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}} \geq km$, as per (69). When $q_{it1} \leq 0$ then constraint (53) becomes binding and q_{it1} is set to 0 (Solution b). Similarly, when $q_{it2} \leq 0$ then constraint (53) becomes binding and q_{it2} is set to 0 (Solution c).

EC.3. Lemma 3

Lemma 3*: Given any combination of betting behaviors as per Lemma 1 and Lemma 2, equilibrium exists, and the equilibrium odd for j=1 is:* $O_{t1} = \frac{\sum_{i\in D1 \cup D2a} p_{it2}(w_{it}+m) + \sum_{i\in D2c}(km)}{\sum_{i\in D1 \cup D2a} p_{it1}(w_{it}+m) + \sum_{i\in D2b}(km)} + 1$

Proof Lemma 3

In IMF, for each object t, the house manipulates the market odds O_{t1} and O_{t2} to establish the equilibrium odds that occur when:

$$O_{t1}Q_{t1} = O_{t2}Q_{t2} \quad (78)$$

Using Lemma 1 and Lemma 2, the LHS and RHS of (78) are:

$$O_{t1}Q_{t1} = O_{t1}\sum_{i\in D1} p_{it1}(w_{it}+m) + O_{t1}\sum_{i\in D2a}(p_{it1}km + a_{it}\frac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}}) + O_{t1}\sum_{i\in D2b}(km) + O_{t1}\sum_{i\in D2c}(0)\text{; and} \quad (79)$$

$$O_{t2}Q_{t2} = O_{t2}\sum_{i\in D1} p_{it2}(w_{it}+m) + O_{t2}\sum_{i\in D2a}(p_{it2}km + a_{it}\frac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}}) + O_{t2}\sum_{i\in D2b}(0) + O_{t2}\sum_{i\in D2c}(km), \quad (80)$$

where all agents $i\in D_1$ that bet per Lemma 1 are defined as $i\in D_1$ and all agents that bet per Lemma 2, solutions a, b and c are defined as $i\in D_{2a}$, $i\in D_{2b}$ and $i\in D_{2c}$, respectively.

On substituting the LHS of (78) with the RHS of (79) and the RHS of (78) with the RHS of (80):

$$O_{t1}\sum_{i\in D1} p_{it1}(w_{it}+m) + O_{t1}\sum_{i\in D2a}(p_{it1}km + a_{it}\frac{p_{it1}O_{t1} - p_{it2}O_{t2}}{O_{t1}O_{t2}}) + O_{t1}\sum_{i\in D2b}(km) + O_{t1}\sum_{i\in D2c}(0) =$$
$$O_{t2}\sum_{i\in D1} p_{it2}(w_{it}+m) + O_{t2}\sum_{i\in D2a}(p_{it2}km + a_{it}\frac{p_{it2}O_{t2} - p_{it1}O_{t1}}{O_{t1}O_{t2}}) + O_{t2}\sum_{i\in D2b}(0) + O_{t2}\sum_{i\in D2c}(km) \quad (81)$$

On Simplifying (81):

$$O_{t1}\sum_{i\in D1} p_{it1}(w_{it}+m) +$$
$$O_{t1}\sum_{i\in D2a}(p_{it1}km) + \sum_{i\in D2a}(a_{it}\frac{p_{it1}O_{t1}}{O_{t2}}) - \sum_{i\in D2a}(a_{it}p_{it2}) + O_{t1}\sum_{i\in D2b}(km) =$$

$$O_{t2}\sum_{i\in D1} p_{it2}(w_{it}+m)+$$

$$O_{t2}\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2a}(a_{it}\frac{p_{it2}O_{t2}}{O_{t1}})-\sum_{i\in D2a}(a_{it}p_{it1})+O_{t2}\sum_{i\in D2c}(km) \tag{82}$$

On Simplifying (82) by substituting O_{t2} for $O_{t1}/(O_{t1}\text{-}1)$:

$$O_{t1}\sum_{i\in D1} p_{it1}(w_{it}+m)+$$

$$O_{t1}\sum_{i\in D2a}(p_{it1}km)+\sum_{i\in D2a}(a_{it}p_{it1}(O_{t1}-1))-\sum_{i\in D2a}(a_{it}p_{it2})+O_{t1}\sum_{i\in D2b}(km)=$$

$$\frac{O_{t1}}{O_{t1}-1}\sum_{i\in D1} p_{it2}(w_{it}+m)+$$

$$\frac{O_{t1}}{O_{t1}-1}\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2a}(a_{it}\frac{p_{it2}}{O_{t1}-1})-\sum_{i\in D2a}(a_{it}p_{it1})+\frac{O_{t1}}{O_{t1}-1}\sum_{i\in D2c}(km) \tag{83}$$

On simplifying (83):

$$O_{t1}(\sum_{i\in D1} p_{it1}(w_{it}+m)+\sum_{i\in D2a}(p_{it1}km)+\sum_{i\in D2a}(a_{it}p_{it1})+\sum_{i\in D2b}(km))-$$

$$\frac{O_{t1}}{O_{t1}-1}(\sum_{i\in D1} p_{it2}(w_{it}+m)+\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2c}(km))\ -\sum_{i\in D2a}(a_{it}\frac{p_{it2}}{O_{t1}-1})=$$

$$\sum_{i\in D2a}(a_{it}p_{it2}) \tag{84}$$

On simplifying (84):

$$(O_{t1}\text{-}1)\ O_{t1}(\sum_{i\in D1} p_{it1}(w_{it}+m)+\sum_{i\in D2a}(p_{it1}km)+\sum_{i\in D2a}(a_{it}p_{it1})+\sum_{i\in D2b}(km))-$$

$$O_{t1}(\sum_{i\in D1} p_{it2}(w_{it}+m)+\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2c}(km))-\ (O_{t1}-1)\sum_{i\in D2a}(a_{it}p_{it2})=$$

$$\sum_{i\in D2a}(a_{it}p_{it2}) \tag{85}$$

On simplifying (85):

$$(O_{t1}\text{-}1)O_{t1}(\sum_{i\in D1} p_{it1}(w_{it}+m)+\sum_{i\in D2a}(p_{it1}km)+\sum_{i\in D2a}(a_{it}p_{it1})+\sum_{i\in D2b}(km))-$$

$$O_{t1}(\sum_{i\in D1} p_{it2}(w_{it}+m)+\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2a}(a_{it}p_{it2})+\sum_{i\in D2c}(km))\ =0 \tag{86}$$

On simplifying (86):

$$(O_{t1}\text{-}1)\ (\sum_{i\in D1} p_{it1}(w_{it}+m)+\sum_{i\in D2a}(p_{it1}km)+\sum_{i\in D2a}(a_{it}p_{it1})+\sum_{i\in D2b}(km))=$$

$$\sum_{i\in D1} p_{it2}(w_{it}+m)+\sum_{i\in D2a}(p_{it2}km)+\sum_{i\in D2a}(a_{it}p_{it2})+\sum_{i\in D2c}(km) \tag{87}$$

On simplifying (87):

$$O_{t1}=\frac{\sum_{i\in D1} p_{it2}(w_{it}+m)+\sum_{i\in D2a} p_{it2}(km+a_{it})+\sum_{i\in D2c}(km)}{\sum_{i\in D1} p_{it1}(w_{it}+m)+\sum_{i\in D2a} p_{it1}(km+a_{it})+\sum_{i\in D2b}(km)}+1 \tag{88}$$

On simplifying (88), note that $a_{it}=w_{it}+m\text{ - }km$:

$$O_{t1} = \frac{\sum_{i\in D1} p_{it2}(w_{it}+m) + \sum_{i\in D2a} p_{it2}(w_{it}+m) + \sum_{i\in D2c}(km)}{\sum_{i\in D1} p_{it1}(w_{it}+m) + \sum_{i\in D2a} p_{it1}(w_{it}+m) + \sum_{i\in D2b}(km)} + 1 \qquad (89)$$

On simplifying (89):

$$O_{t1} = \frac{\sum_{i\in D1\bigcup D2a} p_{it2}(w_{it}+m) + \sum_{i\in D2c}(km)}{\sum_{i\in D1\bigcup D2a} p_{it1}(w_{it}+m) + \sum_{i\in D2b}(km)} + 1 \qquad (90)$$

Since $O_{tj} = 1/P_{itj} \geq 1$ equilibrium odds exist when $\frac{\sum_{i\in D1\bigcup D2a} p_{it2}(w_{it}+m) + \sum_{i\in D2c}(km)}{\sum_{i\in D1\bigcup D2a} p_{it1}(w_{it}+m) + \sum_{i\in D2b}(km)} \geq 0$. Thus, since $0 \leq p_{itj} \leq 1$, $(w_{it} + m) > 0$, and $km > 0$, equilibrium odds exist when agents bet as per Lemma 1 and Lemma 2.

EC.4. Empirical Experiments of Equilibrium Odds

Discontinuous Agent Bets in Odds **-** If agent bets are discontinuous over O_{tj} then the existence of equilibrium odds cannot be guaranteed (Carlsson, et al. 2001). To provide some insights into the utility of IMF in situations such as above, when equilibrium odds might not exist (agents not betting as per Lemma 1 and Lemma 2), we run an experiment with risk neutral agents that bet their entire wealth on only one event j that satisfies $p_{itj}O_{tj} > 1$, i.e., the bets are discontinuous and equilibrium odds do not always exist (verified empirically). The combiner method main effect is evaluated and the results are statistically equivalent to the results in the main experiment.

Existence of Equilibrium Odds **-** Equilibriums odds are defined in IMF as the odds that give $Q_{tj}O_{tj} = Q_t$, and where Q_{tj} and Q_t are functions of O_{tj}. The proof in EC.3 shows that the equilibrium odd O_{t1} is equal to $\frac{\sum_{i\in D1\bigcup D2a} p_{it2}(w_{it}+m) + \sum_{i\in D2c}(km)}{\sum_{i\in D1\bigcup D2a} p_{it1}(w_{it}+m) + \sum_{i\in D2b}(km)} + 1$. However, this can not be used directly to determine the existence of equilibrium odds O_{tj} for a given object t because of the recursive nature of O_{t1} and Q_{t1} (note that q_{itj} is used to determine if $i\in D_1$, $i\in D_{2a}$, $i\in D_{2b}$ or $i\in D_{2c}$, and that the agents use O_{t1} to determine q_{itj}). To empirically validate the existence of equilibrium odds (defined as odds such as $|Q_{tj}O_{tj} - Q_t|/Q_t < 0.0000000001$), we use binary search with $\varepsilon = 0.000000000000001$. Based on this setting, we find equilibrium odds for 98.32% of the objects.

EC.5. Net Benefit and Cost Savings

In a given classification context, Cost Savings (CS) is defined as the difference between the costs that would result if no classification system is used and the costs that results when a classification system is used (Chan et al. 1999). In our fraud context CS is defined as:

CS = P*Fraud Cost - (FN*Fraud Cost + (TP+FP)*Investigation Cost), (91)

where P is the number of fraud instances and Fraud Cost is the cost of one fraud instance.

simplify (91):

CS = (P-FN)*Fraud Cost - (TP+FP)*Investigation Cost (92)

simplify (92):

CS = Fraud Cost*TP - Investigation Cost*(TP+FP) (93)

CS is the same as Net Benefit, since Fraud Cost is the same as FN cost avoidance.

EC.6. Yule's Q

Yule's Q for a pair of base-classifiers is defined as (*ad-bc*)/(*ad+bc*), where *a* is the number of instances where both classifiers are correct, *d* where both are incorrect, *b* where only one of the classifiers is correct and *c* where only the other classifier is correct. A threshold of 0.5 is used to determine whether an agent is correct. Finally, the Yule's Q for a given dataset is calculated as the average of all pairwise Yule's Qs for that particular dataset.

EC.7. Agent Wealth Convergence

Sample plots of the percentage of the agents' wealth to the total wealth in the ensemble over time are presented below. These plots indicate that agents' wealth converge between 250 and 5000 positively classified transactions.

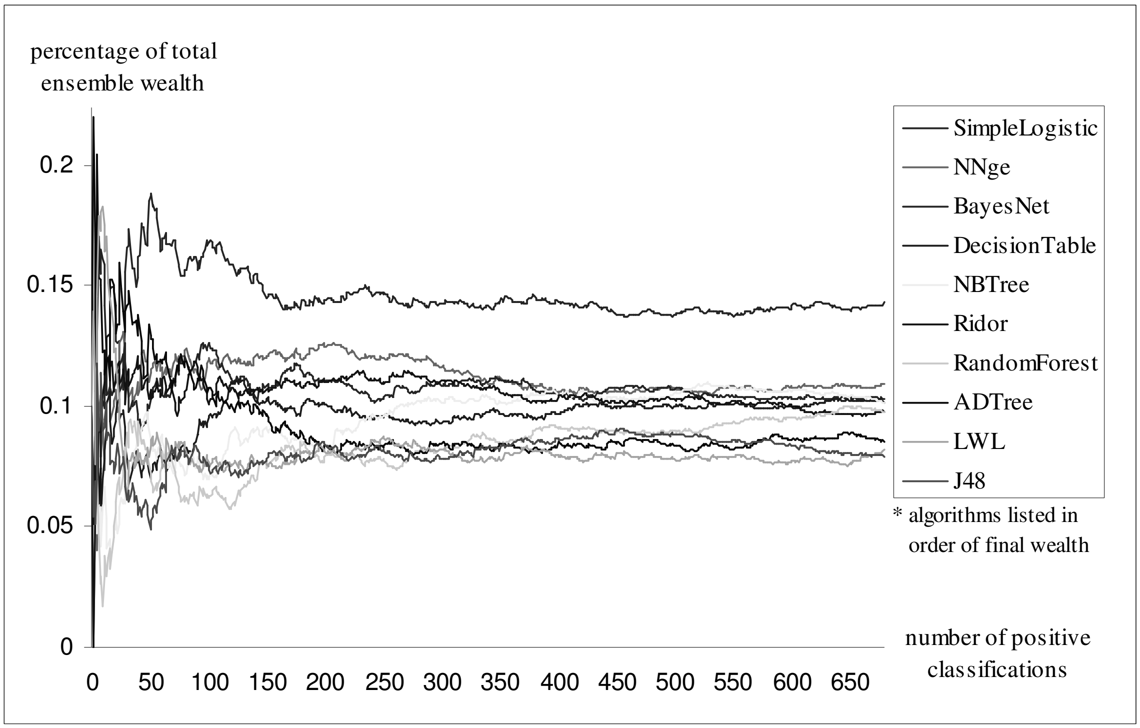

Figure EC-1: Agent Wealth Convergence for Diabetes

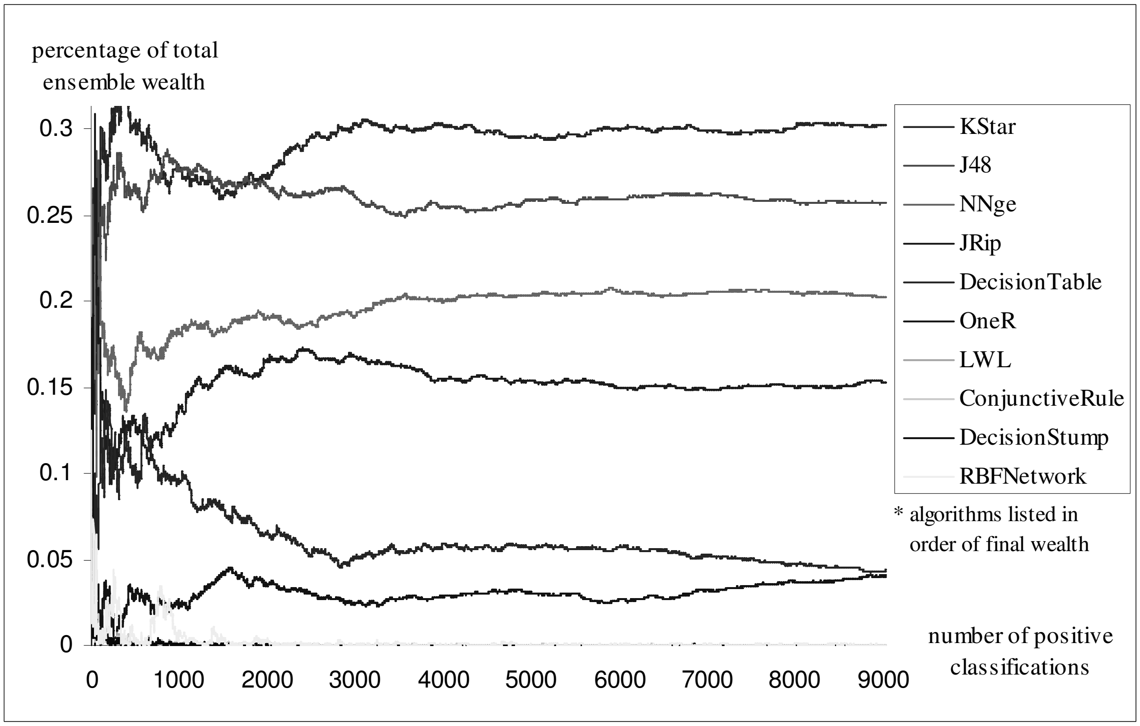

Figure EC-2: Agent Wealth Convergence for COV(1&2)